Calcul

19 avril 2024

On a

$$
\begin{array}{cl}
 & A \subseteq B \\
\iff & \forall x \, (x \in A \implies x \in B) \\
\iff & \forall x \neg \left(\bigwedge \left\{ \begin{array}{c} x \in A \\ \neg x \in B \end{array} \right\} \right) \\
\implies & \forall x \neg \left(\bigwedge \left\{ \begin{array}{c} x \in A \\ \neg x \in B \\ x \in X \end{array} \right\} \right) \\
\iff & \forall x \neg \left(\bigwedge \left\{ \begin{array}{c} \neg\neg x \in A \\ \neg x \in B \\ x \in X \end{array} \right\} \right) \\
\iff & \forall x \neg \left(\bigwedge \left\{ \begin{array}{c} \neg \left(\bigwedge \left\{ \begin{array}{c} \neg x \in A \\ \neg \bot \end{array} \right\} \right) \\ \neg x \in B \\ x \in X \end{array} \right\} \right) \\
\iff & \forall x \neg \left(\bigwedge \left\{ \begin{array}{c} \neg \left(\bigwedge \left\{ \begin{array}{c} \neg x \in A \\ x \in X \end{array} \right\} \right) \\ \neg x \in B \\ x \in X \end{array} \right\} \right) \\
\iff & \forall x \neg \left(\bigwedge \left\{ \begin{array}{c} \neg x \in X \backslash A \\ \left(\bigwedge \left\{ \begin{array}{c} \neg x \in B \\ x \in X \end{array} \right\} \right) \end{array} \right\} \right) \\
\iff & \forall x \neg \left(\bigwedge \left\{ \begin{array}{c} \neg x \in X \backslash A \\ x \in X \backslash B \end{array} \right\} \right) \\
\iff & \forall x \, (x \in X \backslash B \implies x \in X \backslash A) \\
\iff & X \backslash B \subseteq X \backslash A
\end{array}
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