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Masatake Kuranishi (1924–2021)

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Figure 1.

Kuranishi lecturing at the Oka100 conference in Kyoto-Nara, 2010.

Professor Masatake Kuranishi passed away on June 22, 2021. He was one of the truly great mathematicians of the twentieth century, whose legacy permeates whole areas of current mathematics. This memorial article is a tribute to him, with contributions from many of his former students, friends, and colleagues.

Professor Masatake Kuranishi was born on July 19, 1924, in Tokyo, Japan. After an assistantship at Tokyo Tech, he got his doctorate degree in mathematics from Nagoya University in 1951 under the supervision of T. Nakayama. He became an assistant professor at Nagoya University in 1952. From 1954 on, he traveled extensively in the United States, where he held visiting positions at several institutions, including the Institute for Advanced Study, the University of Chicago, the Massachusetts Institute of Technology, and

Princeton University. He was given the rank of professor at Nagoya University in 1957. He settled at Columbia University in 1961, and was the Davies Professor of Mathematics until his retirement in 1999. He received many marks of recognition for his research. These include a Guggenheim Fellowship in 1975–1976, and invitations to speak at the International Congress of Mathematicians in Stockholm in 1962 and in Nice in 1970. He received the Bergman Prize of Wells Fargo and the American Mathematical Society in 2000, and the Geometry Prize from the Mathematical Society of Japan in 2014. Major conferences in his honor were held at Columbia in 1994 and 2005.

When asked about his area of research, Professor Kuranishi would give the short answer of partial differential equations. But this does not accurately reflect his work nor his approach to mathematics, which transcends mathematical disciplines. One can get a sense of this by looking at the book of his selected work, edited by H. Hironaka et al.,[1] or the book by A. Fujiki.[2] Certainly, his major contributions span a very wide range. There is probably a general consensus that they should include:

- His paper on the differentiability of locally compact groups, which was essential to H. Yamabe's eventual solution of Hilbert's 5th problem.
- His theorem on the prolongation of exterior differential systems to determine in a finite number of prolongations whether the system is involutive. The theorem, now known as the Cartan-Kuranishi theory, has important applications in the Lie-Cartan theory of infinite-dimensional Lie groups.

- His contributions to the Kodaira-Spencer-Kuranishi theory of deformations of structures, and particularly his approach to singularities and jumps in the dimension of the moduli space. This is now the basic approach to the moduli space of solutions of many partial differential equations, and complete families are now known as Kuranishi spaces. Early examples include the Atiyah-Hitchin-Singer and Donaldson moduli spaces of self-dual forms and anti-self-dual connections, respectively, with already far-reaching applications to topology and mathematical physics. More and more important examples continue to emerge, most recently from symplectic geometry.

Figure 2.

Kuranishi with Sayuri in the late 1970s.

- His proof in the mid-1960s of the reparametrization invariance of pseudodifferential operators. This paper appears to be unpublished. However, it was known to experts, who taught it in their classes, for example at Princeton. Pseudodifferential operators had emerged at that time from index theory and subelliptic problems, but were still unfamiliar to most people. Their invariance under reparametrizations was an essential building block for microlocal analysis and an ultimate theory of Fourier integral operators.

- His proof of the embeddability of CR structures in dimension ≥ 9. This is a real tour de force, full of difficult estimates whose very formulation is already nontrivial, and which solves contrary to expectation a problem widely considered as out of reach by most experts.

These works and others are discussed in the individual contributions to this memorial article, each with its own perspective. It should be stressed that the list is by no means exhaustive. In particular, Professor Kuranishi had extensive unpublished notes on Fourier integral operators with complex phases, formulated in terms of ideals, so that singularities can be more easily incorporated. In his later years, he developed a theory of what he referred to as Cartan structures, which encompasses Riemannian geometry, conformal geometry, and the Fefferman and Burns-Diederich-Shnider conformal CR structures.[3]

Professor Kuranishi was not just admired for his monumental works in mathematics. He was also the kindest, most generous, and most considerate person that people had ever met. Everyone who knew him loved him.

Figure 3.

2005 Conference at Columbia University celebrating Kuranishi's 80th birthday.

Robert Bryant

When I was a graduate student studying the works of Élie Cartan, I had the great fortune to find Professor Kuranishi's fundamental 1957 paper, "On E. Cartan's prolongation theorem of exterior differential systems," which provided the long-sought capstone in Cartan's geometric theory of partial differential equations.

Figure 4.

Robert Bryant lecturing at the 2005 Columbia Conference in honor of Kuranishi.

Cartan had developed his theory in the late 1890s in order to treat systems of PDE that arose in geometric contexts, particularly systems that were invariant under some Lie group (possibly of so-called “infinite type”) of transformations. Cartan’s basic idea was that one could prove existence of solutions of a (possibly overdetermined or degenerate) system of real-analytic PDE if the system could be “filtered” in such a way that a solution could be built up from initial data by solving a sequence of initial value problems in successively higher numbers of independent variables, each step using the previously found “lower dimensional” solutions as initial data of a Cauchy-Kowalewski system to extend the domain of the partial solution to one higher dimension until one reached the solution in the desired dimension. Cartan called a PDE system that could be filtered in this way “involutive.” (The simplest case of an involutive system is the one encountered in the well-known Frobenius theorem.) Not every PDE system is involutive, of course; for example, the generic overdetermined PDE system has no solutions at all.

The core of Cartan’s approach was a process that Cartan called “prolongation,” by which one could augment a given system of PDE by adjoining derivatives of the unknowns and PDE governing them in such a way that every solution of the original system was a solution of the new system. Cartan believed that, by iterating this process, one would eventually arrive at either a system that had a visible incompatibility (such as a finite relation among the independent variables that literally had no solution) or a system that was involutive. Proceeding on this belief, Cartan attacked and solved an astonishing array of problems in differential geometry, studying Lie transformation groups, Riemannian, conformal, and projective geometry. However, he was never able to prove that his prolongation process actually terminated in one of the two desired ends.

This gap in the theory was finally filled by Professor Kuranishi in the above-mentioned paper, and it was a landmark result. (Its Math Reviews entry starts out, “In this very brilliant paper,…”) He recognized that some regularity condition, which he called “normality,” was needed to prove the finite termination of the process in involutivity and was able to reformulate the problem in such a way that he could reduce it to the Hilbert basis theorem. His approach clarified the relation of Cartan’s notion of “involutivity” with that of regular sequences in local rings and opened the way for major developments in the theory of Lie pseudogroups in the 1960s.

Professor Kuranishi further developed Cartan’s theory in other writings, for me most notably his 1967 São Paolo volume *Lectures on involutive systems of partial differential equations*, which were an epiphany to me as a young mathematician 10 years later. In particular, his rigorous treatment of Cartan’s notion of the “generality” of the space of solutions of a system of PDE clarified many of my misconceptions on the subject and has become an essential tool in my own work to the present day. In fact, it was Professor Kuranishi’s work in this area that gave me the essential ingredients that I needed to tackle the question of the existence and generality of the Riemannian manifolds with exceptional holonomy, a problem that had been open since the thesis of M. Berger in the 1950s.

When I finally had the chance to meet Professor Kuranishi in person, I was gratified to find that he was as gracious and kind as he was brilliant. He was soft spoken but absolutely unafraid to take on the most challenging problems. I will always cherish my memories of our conversations and letters.

Figure 5.

Kuranishi at the 1995 Hayama conference.

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Albert Chau

I was a PhD student at Columbia University during the years 1997–2001. I was fortunate to have Professor Kuranishi as my graduate analysis instructor in the spring term of my first year. It was a standard first-year course leading to one of the qualifying exams, and he focused on function space theory and PDEs. My first impression of Professor Kuranishi was the same as everyone else's. He was patient and warm, genuine and pure, and a true grand master in his field. I knew nothing of his works or even the area he worked in, but I was sure that I wanted to be his student. Unfortunately, I had to leave partway through the term to be with my father who had taken suddenly ill in Hong Kong. I followed my classes through lecture notes mailed to me by my classmates, and I was particularly eager to see what was covered in Professor Kuranishi's class each week. I returned to Columbia at the end of the term, and I was very nervous about final exams and in particular making a good impression on Professor Kuranishi. My reaction to his words at the end of his last lecture was the same as everyone else's. I was relieved and overjoyed! He announced that the final exam was a take home exam which was due the next fall!

My thesis work with Professor Kuranishi began in my third year. I was interested in geometric analysis and related problems. Knowing this, he decided we would read *The formation of singularities in the Ricci flow* by Richard Hamilton, whose office was nearby and who was always happy to talk when we had questions. In our weekly meetings, Professor Kuranishi would listen quietly while writing in his notepad as I lectured. In one meeting, I was explaining how, after rescaling, the flow converges on any compact surface to a metric of constant curvature. As usual, he listened quietly until I finished then he paused for some time before asking "what happens on noncompact manifolds?" I told him I had absolutely no idea, and that I would look into it. After experimenting with the flow of complete asymptotically hyperbolic models for a few weeks, I told him of my results. He again paused for some time before asking "what about general Kahler manifolds?" I did not know it then, but with these two simple questions Kuranishi steered me directly onto the path leading to my thesis and graduation, and would continue to guide my research for many years to follow!

I am forever grateful for Professor Kuranishi's support and advice, which served me well for so long. His knowledge and wisdom were matched only by his generosity and understanding. Being his student was an honor I will carry with me for life.

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Elisha Falbel

Masatake Kuranishi was my advisor at Columbia University in the second half of the 1980s. The story of how I got there is interesting with respect to Kuranishi's relation to Brazilian mathematics.

Kuranishi travelled twice to Brazil. He was invited by Alexandre Martins Rodrigues, who did his thesis in Chicago under S.-S. Chern in 1957, the year of the publication of Kuranishi's important paper on Cartan's prolongation theorem. Since Kuranishi was at Chicago then, this may have been the start of their collaboration, which became more intense in 1961 when Kuranishi became a professor at Columbia. Subsequently, Rodrigues went to Columbia and they wrote a paper applying Cartan-Kuranishi's prolongation theorem to pseudogroups.

In 1965, it was Kuranishi's turn to spend several months teaching about involutive systems. I was surprised, preparing this account, that his publication *Lectures on involutive systems of partial differential equations*[4] from January 1967 is not included on MathSciNet. After this first visit, pseudogroups and infinite Lie groups were a constant theme of the Mathematics Institut of São Paulo.

Kuranishi visited São Paulo a second time in 1981, again staying for several months. Kuranishi had been studying CR structures for several years and had solved the difficult local embedding problem for strictly pseudoconvex of higher dimensional structures. He gave a class on the local embedding theorem for CR structures.

When I showed an interest in CR structures, Alexandre Rodrigues wrote to Kuranishi asking if I could be his student at Columbia. He agreed immediately, a mark of confidence in his Brazilian friend. I remember my first meeting with him as very frustrating for both of us as I couldn't understand his English. I told other graduate students about it and they reassured me that I would quickly get used to his Japanese accent. Indeed that was the case. In our weekly meetings, Kuranishi never imposed a research path but always tried to help when technical difficulties appeared. The feeling of total liberty was overwhelming and I will never forget one day when I thought the thesis was not advancing and, by chance, we crossed each other on the street and he just told me that the idea I had had last month was very good and that I should write a thesis based on it. This was exactly what I needed to hear.

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Charles Fefferman

Figure 6.

Charles Fefferman lecturing at the 2005 Columbia Conference in honor of Kuranishi.

I'd like to illustrate Kuranishi's qualities by recalling my experience as a referee of his monumental paper on CR embeddings.

The result is fundamental and the obstacles daunting. Kuranishi brought in a family of weighted norms with singularities not present in the hoped-for embedding. If the embedding existed, he could use it to prove his weighted estimates. If the estimates held, he could use them to correct an approximate embedding and produce a better approximate embedding. To avoid circular reasoning, the whole process was wrapped inside a Nash-Moser iteration.

My first reaction was that Kuranishi was taking a huge gamble. Why introduce extra singularities, hoping eventually to get rid of them? My heart sank as I discovered a fatal error in Kuranishi's proof. Indeed, I thought, the plan was doomed.

Some time later (several months? I no longer remember), Kuranishi came back with a new proof, based on the same ideas, but with major changes to avoid the fatal error. This time, the proof was 100% correct. To this day I don't understand why it ever had a chance, but I read it line by line with immense care, and it works. I think his paper on CR embeddings reflects Kuranishi's personality, combining gentle patience, immense courage, and deep thought. Let me just add that I never heard Kuranishi say an unkind word. Kuranishi was a rare soul.

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Robert Friedman, John Morgan

Masatake Kuranishi was recruited to Columbia by Sammy Eilenberg and Ellis Kolchin in 1961. His fundamental paper on deformations of complex structure had not yet appeared in the *Annals of Mathematics*, but it was widely known. Columbia scored a major coup by attracting Kuranishi, especially since he was actively courted by many other top departments. Beyond the many strengths of the Columbia Mathematics Department, Kuranishi may have been influenced by the personal connection of his wife's family to the university.

It has been known since Riemann that the complex structure on a compact Riemann surface depends locally on complex parameters, and work of Kodaira-Spencer generalized this to every compact complex manifold M under the technical hypothesis that the sheaf cohomology group $H^2(M; T_M) = 0$, where T_M is the sheaf of holomorphic tangent vectors on M. In fact, their pioneering work extended to the study of varying the complex structure in many different contexts. For example, the complex structure on a holomorphic vector bundle over a fixed compact complex manifold, but always under the assumption similar to the vanishing of $H^2(M; T_M)$: a space of "obstructions" vanishes. However, it was by this time well-known that there are examples where the change in complex structure is described locally not by an open set in $\mathbb{C}^n$, but rather by a closed analytic space inside such an open set, defined by the vanishing of a finite number of holomorphic functions. In his fundamental paper "On the locally complete families of complex analytic structures,"[5] Kuranishi established the general case of the Kodaira-Spencer result. The paper "New proof for the existence of locally complete families of complex structures"[6] reproves this result, and gives an elegant, flexible, and very general method for attacking such questions which is not limited to complex geometry. The key idea is to consider a C^∞ or holomorphic map $F: U_1 \to U_2$, where $U_1 \subseteq V_1$ and $U_2 \subseteq V_2$ are open neighborhoods of the origin in real or complex Banach spaces V_1, V_2, such that $F(0) = 0$ and the differential dF is Fredholm. Then, possibly after shrinking U_1 and U_2, the space $F^{-1}(0)$ is modeled on a finite-dimensional space, where V_1 and V_2 are replaced by finite-dimensional vector spaces. In particular, if F is holomorphic, then $F^{-1}(0)$ is locally modeled on a finite-dimensional analytic space. Kuranishi's method applies to all of the deformation theory problems studied by Kodaira-Spencer, for example to the deformation theory of holomorphic vector bundles on a compact complex manifold.

The work of Kuranishi has applications far beyond the realm of complex geometry. The first application to gauge theory that we know of appears in Atiyah-Hitchin-Singer, "Self-duality in four-dimensional Riemannian geometry"[7]: they show that the moduli space of irreducible self-dual connections on a self-dual 4-manifold with positive scalar curvature is a smooth manifold, by using the Banach space methods pioneered by Kuranishi. Similar arguments were used by Donaldson shortly thereafter in his fundamental work on smooth definite 4-manifolds,[8] and in subsequent work constructing smooth invariants of 4-manifolds. In all of these cases, the obstruction space vanishes, either by the assumption of positive scalar curvature or by choosing a generic metric and using Sard's theorem. A case where the full power of Kuranishi's method is needed, and which unites the strains of complex geometry and gauge theory, is the following: by another fundamental result of Donaldson, for a Kähler surface S, the moduli space of anti-self-dual connections on S of a fixed topological type is identified with the moduli space of holomorphic structures on the corresponding complex vector bundle which are stable in the sense of Mumford-Takemoto.[9] This result was generalized to smooth projective varieties of any dimension by Donaldson, and to compact Kähler manifolds in general by Uhlenbeck-Yau. One can show that the local models for the moduli space described above are diffeomorphic in the appropriate sense. A Kähler metric is far from being a generic metric, and the corresponding holomorphic structure may well have a nonzero obstruction space, so that the full power of Kuranishi's method is needed.

Figure 7.

P. Griffiths, J. Morgan, and Y.-T. Siu at the banquet of the 2005 Columbia Conference in honor of Kuranishi.

Beyond its applications to gauge theory, the method of Kuranishi has had many other applications, for example in the construction of Gromov-Witten invariants by defining virtual fundamental cycles on general symplectic manifolds (see for example Li-Tian, "Virtual moduli cycles and Gromov-Witten invariants of general symplectic manifolds"[10]). More recently, Fukaya and Ono have defined the notion of Kuranishi structures in symplectic geometry,[11] with related ideas and refinements appearing in unpublished work of Joyce and more recently in the work of McDuff-Wehrheim.[12] These examples serve to demonstrate the power and continued relevance of the "Kuranishi method" or the use of "Kuranishi models," as Kuranishi's profound and original work is now called.

On a personal note, the two of us began working on gauge theory and its implications for complex surfaces when we were fortuitously both at MSRI in 1985. As we struggled to understand Donaldson's ideas and their amazing implications for the smooth topology of complex surfaces, it was a great privilege for us to be able to ask Kuranishi about his work on deformation of complex structures and the "Kuranishi method" more generally. His patient and careful explanations of his published work and his unpublished extensions were both invaluable and inspirational.

Kuranishi was a modest and gracious colleague and was unfailingly polite and soft-spoken. One would never have known, watching him around the department, that one was in the presence of such a mathematical giant. When it was his turn to be department chair, he willingly took on the task and worked hard at being a good chair. He led the department well, in spite of the daunting cultural differences he had to overcome when dealing with the university administration.

We were all fortunate to have had him as a life-long colleague and friend.

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Akito Futaki

I met Professor Kuranishi for the first time in the early 1990s at Tokyo Institute of Technology (Tokyo Tech for short) where I was a faculty member during 1989–2012. He was spending his summer vacation every year in Japan as a visitor of Tokyo Tech. He is of course famous for the deformation theory of complex structures and CR geometry. Around 1990, he was well-known because of the role of Kuranishi's method in describing the moduli space of the solutions of self-dual equations on 4-manifolds, that is, Donaldson theory. But at that time, Kuranishi himself was engaged in the Hopf product conjecture which states that there is no Riemannian metric of positive sectional curvature on $S^2 \times S^2$. This conjecture is still open. He was trying to disprove the conjecture by constructing a metric of positive sectional curvature. It was a hot topic among us that Prof. Kuranishi was trying to construct such a metric, and some of our colleagues were following his computations. When we had a chance to have dinner with his wife, she complained that her husband stayed up late struggling with computations. Kuranishi's computations can be found in the paper "On some metrics on $S^2 \times S^2$" published in Proc. Symp. Pure Math. 1993, which concludes with some problems mentioning "These are tantalizing questions which remain to be answered."

Tokyo Tech was a good place for him to visit because of the connection between Tokyo Tech and his father and himself. According to an essay which he wrote in Japanese, his father graduated from Tokyo Imperial University (currently the University of Tokyo), worked for the army related to aircraft design, was dispatched to Germany for one year, and then became a professor at Tokyo Tech in 1939. After World War II ended in 1945, his father was purged from the university because he had worked for the army. He lost his job for seven years, but in 1952 he became a full professor at Nihon University, a private university in Tokyo. Kuranishi himself got a position as an assistant at Tokyo Tech after graduating from Nagoya University in 1947. Kuranishi's first paper was published by *Kodai Mathematical Seminar Reports*, volume 1, 1949 (currently issued as *Kodai Mathematical Journal*). "Kodai" is a Japanese nickname for Tokyo Tech as it is an abbreviation of the Japanese name Tokyo Kogyo Daigaku, and nowadays the nickname "To-Ko-Dai" is more commonly used. He moved to Nagoya University in 1950 as an assistant, was promoted there to lecturer in 1951, and obtained a PhD degree in 1952. Then he was promoted to assistant professor in 1952, to full professor in 1957, and moved to Columbia University in 1961.

As a personal memory, in the 90s the geometers in the Tokyo area used to hold workshops in rural areas outside Tokyo. In one such workshop we had an excursion on foot, but Kuranishi walked so fast that I could not catch up with him. He was in his late 60s and was 30 years older than I. In March 2010, I visited Columbia University and shared his office. He invited me to lunch, but I had to decline because he had trouble walking.

Kuranishi's method remains a basic idea to describe moduli spaces of solutions of many nonlinear geometric PDEs. Thank you, Professor Kuranishi!

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Phillip Griffiths

Aside from occasional conferences and mutual attendance at lectures, my main contacts with Masatake Kuranishi have been through his work. Although I was generally familiar with his very deep proofs of the Cartan-Kuranishi prolongation theorem, the CR-embedding theorem, and a few other works, by far what I know best is his work on deformation theory. This is the formulation and proof of the fundamental result, the existence of the Kuranishi space. As a student of Don Spencer in the early 1960s, I was quite interested in the theory of deformations of compact, complex manifolds that had then just recently been created by Kodaira and Spencer, and the construction of the Kuranishi space was one of the crowning achievements of the subject. I will try to briefly and informally explain this result.

Figure 8.

Phillip Griffiths lecturing at the 2005 Columbia Conference in honor of Kuranishi.

A basic feature of compact complex manifolds X, and more generally of complex analytic varieties, is that they usually occur in families. The classic example is given by the 1-dimensional complex tori

$$E_\tau = \mathbb{C}/\mathbb{Z} + \tau\mathbb{Z}$$

where τ is a point in the upper half plane. Using doubly periodic functions, E_τ is realized as a smooth cubic curve in the projective plane with the affine equation

$$y^2 = x^3 - ax - b$$

in $\mathbb{C}^2$. Looked at either way, the complex manifold E_τ depends on a parameter, either τ or algebraically using $j = \frac{1728u^3}{a^3 - 27b^2}$.

It is due to Riemann that a compact Riemann surface X of genus g depends on $3g - 3 + \rho$, $\rho = \dim \mathrm{Aut}(X)$, parameters, or moduli. Moreover the *dual* of the tangent space to the family of eigenvalence classes of X's is given by the space

$$(1) \quad H^0\left(X, K_X^{\otimes 2}\right)$$

of quadratic differentials on X; these are global objects whose local expression is $f(z)dz^2$ where z is a holomorphic coordinate on X. Mathematicians had unsuccessfully searched for the analogue of the dual of (1) in higher dimensions; i.e., for an object that could serve as the expected tangent space to the family of (equivalence classes of) X's. In particular this would suggest an answer to the question: How many parameters does the complex structure of X depend on? The breakthrough came in the early 1950s when Kodaira and Spencer realized that the recently established Kodaira-Serre duality theorem gives the expression

$$(2) \quad H^1(X, \Theta_X) \cong H^0(X, K_X)^*$$

for the tangent space to the parameter space for a compact Riemann surface. Here the left-hand side of (2) is the first cohomology group of the tangent sheaf Θ_X. This suggested that given a class in $H^1(X, \Theta_X)$ there should be a prescription to construct at least a first order deformation X_t of the complex structure of X.

As with de Rham's theorem representing the topological cohomology of a manifold by differential forms, the cohomology group $H^1(X, \Theta_X)$ is represented by differential forms φ that in local holomorphic coordinates z^i are

$$\varphi = \sum_{i,j} \varphi^i_{\bar{j}} \frac{\partial}{\partial z^i} \otimes d\bar{z}^j$$

where the $\varphi^i_{\bar{j}}$ are C^1 functions. The condition that such φ represent a cohomology class is

$$(3) \quad \bar{\partial}\varphi := \sum_{i,j,k} \frac{\partial \varphi^i_{\bar{j}}}{\partial \bar{z}^k} \left(\frac{\partial}{\partial z^i} \otimes d\bar{z}^j \wedge d\bar{z}^j \right) = 0.$$

Then the differentials of the holomorphic coordinates on X_t are locally spanned by

$$\omega^i := dz^i + t\left(\sum_j \varphi^i_{\bar{j}} d\bar{z}^j\right).$$

The coefficient of t in the Frobenius integrability condition

(4) $d\omega^i \equiv 0 \text{ modulo } \{\omega^1, \dots, \omega^n\}$

is just $\bar{\partial}\varphi = 0$.

When $\dim X = 1$ the conditions (3) and (4) are automatic and this gives a family of complex structures on X parametrized by an open neighborhood $\mathrm{Def}(X)$ of the origin in $H^1(X, \Theta_X)$. However when $\dim X = 2$ the coefficient of t^2 in the integrability condition (4) is not automatically satisfied. To have this there must be a solution to the equation

(5) $[\varphi, \varphi] = \bar{\partial}\varphi.$

The left-hand side of (5) is a cohomology class in $H^2(X, \Theta_X)$, and a basic existence result of Kodaira-Nirenberg-Spencer was that when

(6) $H^2(X, \Theta_X) = 0$

there is a $h^1(\Theta_X) := \dim H^1(X, \Theta)$ dimensional family of complex structures on the differential manifold X, as in the Riemann surface case.

An additional feature is that if (6) is satisfied, then the family parametrized by $\mathrm{Def}(X)$ is *versal* in the sense that any local family of deformations of X with a parameter space S is, after passing to a finite cover of S, induced by a mapping $S \to \mathrm{Def}(X)$. The condition (6) is generally not satisfied, and even if it is the obstruction equation may or may not be nontrivial. This is where the situation stood before Kuranishi. In a work that is in all ways a tour de force (a characteristic of the proofs of his major results), stated very informally, Kuranishi proved:

Theorem.

Given a compact complex manifold X, there exists a Kuranishi space $\mathrm{Def}(X)$ with the properties

(i) $\mathrm{Def}(X)$ is an analytic subvariety of $H^1(X, \Theta_X)$;

(ii) $\dim \mathrm{Def}(X) \geq h^1(\Theta_X) - h^2(\Theta_X)$;

(iii) there is a versal deformation $\mathcal{X} \to \mathrm{Def}(X)$ of X;

(iv) the natural action of $\mathrm{Aut}(X)$ on $H^1(X, \Theta)$ preserves $\mathrm{Def}(X)$.

One kernel of the idea of Kuranishi's construction is the following: Given a Θ_X-valued (0,1) form φ_1 as above, one tries to find a formal series

$$\varphi(t) = \varphi_1 t + \varphi_2 t^2 + \cdots$$

that if convergent would give a 1-parameter deformation X_t of $X = X_0$. The Frobenius integrability condition (4) then is equivalent to the Maurer-Cartan equation

$$\bar{\partial}\varphi(t) + [\varphi(t), \varphi(t)] = 0$$

of which the first two terms are

$$\begin{aligned} \bar{\partial}\varphi_1 &= 0 \\ \bar{\partial}\varphi_2 + [\varphi_1, \varphi_2] &= 0 \\ &\vdots \end{aligned}$$

Given a metric on X, every class in $H^1(X, \Theta_X)$ has a unique harmonic representative. Kuranishi's idea was to use these representatives, and then take the solution space to the equation

$$(7) \quad \bar{\partial}\varphi + [\varphi, \varphi] = 0$$

as defining a candidate for $\mathrm{Def}(X)$. Kuranishi's proof that this actually works is a masterpiece of mathematical argument. One aspect is that although nonlinear, equation (7) is not too nonlinear. On the other hand, an indication of the complexity of the argument is that it is known by example (Vakil) that the analytic variety $\mathrm{Def}(X)$ can have arbitrarily nasty properties —singular, everywhere nonreduced, etc.

A corollary of the result (or rather of its proof) is the estimate

$$\left\{ \begin{array}{c} \text{number of parameters} \\ \text{of the complex} \\ \text{structure on } X \end{array} \right\} \geq h^1(\Theta_X) - h^1(\Theta_X),$$

which provides the best possible answer to the question stated above.

Following the pioneering work of Kodaira-Spencer-Kuranishi the subject of deformation theory exploded. There is a deformation theory of almost every-thing; schemes, morphisms, algebras,…to list just a few. As an example, the deformation theory of isolated singularities of analytic varieties is of particular interest, one result being an analogue due to Grauert of Kuranishi's theorem, which stands as the centerpiece of local deformation theory.

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Joseph J. Kohn

I first met Kuranishi in the 1950s while I was a graduate student in Princeton. He was an active participant in Spencer's "Nothing Seminar," which often included Kodaira, Calabi, Grauert, and Hirzebruch as well as graduate students. We were all amazed by his breadth of knowledge, his clarity, and his brilliance. For me, in particular, he was a source of inspiration, a role model.

Figure 9.

Kuranishi with J. J. Kohn and L. Nirenberg at a conference in Prague on Several Complex Variables.

Kuranishi's construction of locally complete deformations of compact complex manifolds is a tour de force: the jewel on top of the crown of the Kodaira-Spencer deformation theory. His elegant proof of the invariance of pseudodifferential operators under diffeomorphisms is still another example of his ability to get at the heart of the matter.

Louis Boutet de Monvel proved a local embedding theorem for strongly pseudoconvex manifolds of complex dimension greater than 2 with an ingenious application of the $\bar{\partial}$-Neumann problem. The natural question that arose was to prove an embedding theorem for CR manifolds. This would require generalizing the $\bar{\partial}$-Neumann problem to domains with boundary contained in CR manifolds. This generalization seemed impossible since it involved calculations with degenerate vector fields. Kuranishi was fascinated and challenged by this problem and attacked it with great originality, vigor, and remarkably powerful technical skill. Over the years, I marveled at his steadfast perseverance as he carried out one promising approach after another. He did not give up and finally found a brilliant and extremely intricate solution to the problem in all real dimensions greater than 7. (Kuranishi's method was later sharpened by Akahori to prove the result in dimension 7; Nirenberg found a counterexample in dimension 3; the problem remains unsolved in dimension 5.)

Additionally he has made many other stellar contributions: the Cartan-Kuranishi theorem, involutive systems of partial differential equations, etc. Apart from his research, he was an outstanding expositor, lecturer, and mentor.

My admiration of Kuranishi is not limited to his brilliant mathematical career. In particular I admired his integrity and his modesty. He was a very generous host introducing my wife and me to the finest Japanese cuisine both in special restaurants, exclusive clubs, and in his home.

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Ngaiming Mok

Professor Masatake Kuranishi was such a kind, helpful, and generous person. We all miss him. I had the good fortune of having met Masatake in the early years of my career while I was working at Princeton. Later in 1984, I was invited to give a talk at Columbia University, and Masatake was among those who were keen to recruit me there. Soon I accepted an offer from Columbia and was going to take up the job in fall 1985. In the same year, 1984, I was awarded a Sloan Fellowship and I decided to spend spring 1985 in Paris, which made my move from Princeton to Columbia logistically cumbersome. Masatake offered to help me by storing some of my luggage. At the end of 1984, Patrick, a good friend of mine at Teachers' College, drove me to New York City to put some luggage in Masatake's apartment, and we had pleasant conversations with him. Both Patrick and I were interested in Japanese culture, and Masatake was a striking example of a cultured Japanese scholar in the classical tradition who was interested in and who knew quite a bit about Chinese culture. This was the beginning of my interactions with Masatake, through which we came to know each other culturally.

Figure 10.

Kuranishi with Ngaiming Mok at Kuranishi's 80th Birthday Conference in Columbia in 2005.

Masatake was chair of the Department of Mathematics at Columbia when Julia and I moved there in 1985, before the fall semester started. With the help of Masatake and Professor Phong we were able to move into an apartment along Broadway located very close to the campus. The apartment was very nice except that we were not quite used to the noise level. In the spring of 1985, Julia gave birth to our daughter Vivienne, and as summer was approaching, the task of taking care of the baby coupled with the heat and noise became rather daunting. Masatake and his wife Sayuri offered to let us move into their apartment along Riverside Drive, which was much quieter, when they took their summer break in Japan. It was this way that our first summer in New York City left us with a very pleasant memory and we were so thankful to Masatake and Sayuri.

While at Columbia, I was already working on bounded symmetric domains, which interested Masatake from the point of view of CR geometry, where he had made fundamental contributions on the local embeddability of strongly pseudoconvex CR manifolds of dimension ≥ 9. We had exchanges on strictly pseudoconvex CR manifolds of higher codimension associated to bounded symmetric domains of rank ≥ 2. On top of mathematics, I also had the good fortune of having cultural exchanges with Masatake on China and Japan. While this started from the very beginning at Columbia, in 1987 I got more motivated as I was invited by the Hironaka Foundation to make a trip to Japan to visit the University of Tokyo, Osaka University, and Tohoku University. I suspected that Masatake might have made a recommendation for the invitation but in any event he was very happy to get me prepared culturally for my first trip to Japan. I spoke frequently to him in Japanese so that when I went to Japan, I was confident enough to have dialogues with my hosts in Japanese. Masatake was interested in the Chinese classics and he gave me a book which was a Japanese translation, in the classical style, of works of Zhuangzi (369–286 BC), a contemporary of Mensius and the most famous Taoist of the Warrior Period. Like probably many Japanese scholars of his generation, Masatake had a liking for Chinese poetry, and he told me that certain Chinese poems were famous in Japan, and some were perhaps more so than in China, such as the poem in the Tang Dynasty written by Zhang Ji on the Hanshan Temple in Suzhou. I also learned from him about the legendary Japanese monk Kukai (774–835) who travelled with an expedition to China and was originally credited with the writing of the famous Iroha poem, and of the versatile Chinese monk Jian-Zhen (Ganjin in Japanese, 688–763) who accepted an invitation to teach Buddhism in Japan, became blind due to the sea journey, and influenced Japan culturally through both Buddhism and architecture. I learned a lot from Masatake on Japanese culture, was at ease when we chatted about China and Japan, and felt enlightened to be in his company.

In the early 1990s, while I was working in Paris, Masatake came to visit me. I invited him to dinner with my family in the suburbs. Courteous as always, he brought us a bottle of Sauvignon blanc and smoked ham from the local charcuterie. We had an enjoyable evening, and I had the impression that he agreed with me that Paris was a good place to bring up children. It was somewhat unexpected that I had an invitation to take up a chair professorship at the University of Hong Kong (HKU) in 1994, which I accepted. In the summer two years later, I organized the first-time conference at HKU on "Aspects of Mathematics: Algebra, Geometry and Several Complex Variables" and invited Masatake for a research visit including giving a lecture at the conference. He delivered a lecture entitled "Some remarks on bounded symmetric domains" and contributed an article "CR structures and bounded symmetric domains" to the proceedings of the conference, proposing that Shilov boundaries of certain irreducible bounded symmetric domains of rank

≥ 2 could be the source of a rich theory for strongly pseudoconvex CR structures of higher codimension on par with the role of the boundary of the complex unit ball as a strongly pseudoconvex CR manifold of codimension 1. This is indeed an interesting direction of research that has yet to be systematically explored. As Masatake was staying for a longer time, we had the pleasure of taking him to Hong Kong Park. His characteristic childlike smile brightened our day, and we have kept some precious pictures of the joyful trip. When Sayuri joined him later from Japan she brought Vivienne a kimono as a gift, which my daughter wore on her birthday every year until it became too small for her. Sayuri and Masatake insisted on inviting us to the famous Japanese restaurant Nadaman and we had a sumptuous dinner and an enjoyable evening with them.

In 2005, a big conference on “Complex Analysis, Differential Geometry and Partial Differential Equations” was held at Columbia University celebrating the 80th birthday of Masatake. I was happy to have been invited back to Columbia for a stay and to give a lecture at a conference paying tribute to Masatake for his accomplishments across different areas in mathematics. It was attended by a large and very representative group of experts from around the world in a wide range of research fields, which testified to the leadership role Masatake had been playing and the popularity he enjoyed in the mathematical community. That year Vivienne was spending one year of her undergraduate study in New York City. Julia, Vivienne, and I paid a visit to Sayuri and Masatake in their home along Riverside Drive, and we had a chance to go over some photo albums of theirs. We shared the happy memories of the time we spent together in the late 1980s, and of the summer when the three of us were living in their apartment while they were on vacation in Japan. During my stay at Columbia, as courteous hosts Sayuri and Masatake also treated us and some close friends to dinner at a Chinese restaurant. I have a delightful memory of the joyful stay we had in New York City.

After Sayuri passed away, I heard that Masatake retired in Japan, staying in Yokohama. Every now and then I learned of news on Masatake from my academic friends in Japan, in particular the news that he had appeared in the early years of his retirement in activities of the Japanese Mathematical Society. Regretfully I did not make a trip to Japan to pay him a visit in Yokohama. Masatake was a person of great integrity. He was straightforward, down-to-earth, and very humble, and he demonstrated remarkable perseverance both in his mathematical endeavors and in his everyday life. The fond memories of Masatake as a wonderful teacher, a classical scholar, and as a person of great humanity will always stay in my mind.

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Shigefumi Mori

I would like to join many friends in expressing sincere condolences on Professor Masatake Kuranishi's death.

In early 1985, I received an international phone call at Nagoya from him. My research style in the early 1980s was to change places of research between Japan and the United States occasionally, since it helped me to get inspiration and to concentrate. At the time, I was excited to do computations related to the minimal model program and wanted to concentrate on it more.

Among the Mathematics faculty at Columbia University, Bob Friedman was a friend of mine since our stay at Harvard around 1980, and Henry Pinkham became a close friend through our correspondence on 3-dimensional terminal singularities. With its strong algebraic geometry group including Boris Moishezon and Nick Shepherd-Barron, Columbia University was an attractive place for me to visit, but I was very slow to make decisions, as usual. Kuranishi's phone call as chair of mathematics was the final push for my decision to visit Columbia University. Since I had admired his mathematics, his personal encouragement and official invitation meant a lot to me.

In September 1985, Reiko and I and our two kids settled in a flat at 533 W 112 Street near Columbia University, and stayed there for two years. Professor and Mrs. Kuranishi, I mean, Masatake and Sayuri, were always very nice to us; they often invited us to dinner at their home. I remember that Sayuri talked cheerfully and Masatake listened to her smilingly. We enjoyed pleasant conversations and the nice view of the Hudson River. They even lent us some of their Japanese ukiyo-e pictures to hang on the tasteless white wall of our home. I could soon start making computations while giving calculus courses.

Sometime after I arrived, I learned that Masatake had been badly injured in a traffic accident in Europe in 1983 and had difficulty walking. I had not noticed it since he had almost overcome it through many rehabilitation walks. I was amazed at his perseverance. Once Masatake wrote that he might not always learn new mathematics by interacting with mathematicians or attending a symposium, but it was rewarding and stimulating for him to watch others try hard and obtain new results. His warm words comforted those of us who were indebted to Masatake and Sayuri.

During my second year 1986/87, the Mathematics Department organized a special year in algebraic geometry; we had quite a few visitors including A. Beauville, F. Catanese, R. Lazarasfeld, E. Looijenga, Y. Miyaoka, D. Morrison, L. Szpiro, (I might have missed someone), and an occasional visitor, J. Kollár, who was a visitor at IAS Princeton. It was like a reunion since I had met many of them earlier either at Harvard or IAS. One day, during a conversation with János, I noticed that I could use a result of Y. Kawamata's in my research. I could settle the existence of 3-dimensional flips by additional computations over several months and complete the 3-dimensional minimal model program. It led to my receiving the Fields Medal at ICM Kyoto 1990.

I am wholeheartedly thankful to Professor Kuranishi for having offered us the opportunity to come to Columbia University, which was rewarding to my family as well as me.

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Makoto Namba

I became a graduate student at Columbia in the autumn of 1967. I had difficulty speaking and understanding English from the start. When all of the students laughed at the jokes of our teachers, only I couldn't laugh. In such lonely days, I kept my mind stable by the kindness of Kuranishi and his wife Sayuri. I was invited to dinners at their apartment many times, not only with other guests but also alone. Kuranishi looked relaxed with a pipe. He laughed cheerfully and didn't look like he was worried about solving difficult problems in mathematics. But, one evening, Sayuri whispered to me that Masatake was working very hard.

One day in 1969, after returning from Montreal University to Columbia University, Professor Masatake Kuranishi asked me to check the first draft of his Montreal lecture notes "M. Kuranishi: Deformations of Compact Complex

Manifolds."[13] It was the first time that I read something that Kuranishi had written on mathematics. I devoted all my energy to reading it. I was deeply moved and influenced by reading it. Later in 1971, using the idea in the lecture notes, I wrote my thesis under Professor Kuranishi and got a PhD degree from Columbia University. Looking back later, the content of the lecture notes was a detailed explanation of his paper "M. Kuranishi: New proof for the existence of locally complete families of complex structures,"[14] and was characteristic of his work. That is, Kuranishi did calculations with a skill all his own, for the goals which looked apparently simple and easy to attain, but in fact were terribly difficult. He looked as if he was a mountain climber, solely challenging unclimbed peaks.

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Junjiro Noguchi

My personal communication with Professor Masatake Kuranishi started on the occasion of the Osaka International Conference on Complex Geometry and Related Topics, Dec. 1990 in Osaka, chaired by Shingo Murakami. Since then he periodically came to the weekly seminar on complex analysis and geometry at Tokyo Institute of Technology in the summer, and later from 1998 at the University of Tokyo, Komaba. He owned an apartment at Yokohama City, south of Tokyo, and stayed there during summer breaks from New York. He phoned my home to inquire about seminars and workshops, since he did not use email very much. Sometime after his retirement from Columbia University in 1999, he moved from New York to Yokohama, and then he became a regular member of our seminar. Besides the weekly seminar he was always an important participant at our meetings in Japan, e.g., Geometric Complex Analysis in Hayama 1995, Oka 100 in Kyoto/Nara 2001, Hayama Symposium, Winter Seminar on Several Complex Variables, etc.

Figure 11.

Kuranishi with D. Burns, T. Ohsawa, and J. Noguchi.

My research topics were a bit different from those of Professor Kuranishi, but we shared a common interest in complex analysis. Because of his deep and broad interest in mathematics, I enjoyed discussing mathematics with him; it was a real pleasure to hear his comments and talks. His presence alone activated and motivated the seminars and the meetings very much. His straightforward but friendly questions were appreciated a lot by the speakers and all participants of the seminars/meetings. We sometimes went with seminar participants to drink and eat at Izakaya (a sort of pub in the Japanese style) together. It was always great fun for all to chat together surrounding Professor Kuranishi. Afterwards, we shared a train going home, since our homes were in the same direction. On one day of March 1998, I fortunately had a chance to visit his apartment in New York with Professor Hirotaka Fujimoto. He and his wife welcomed us with Sukiyaki. It was a delight for me to find a pair of KEF monitor speakers of the bookshelf type in his room, knowing he had the same hobby as me. I enjoyed the beautiful view of the Hudson River from the apartment as referred to by many others.

I would like to tell two episodes about Professor Kuranishi. According to him,[15] his paper related to Hilbert's 5th problem (then he was an assistant at Tokyo Institute of Technology) was brought to America by Shizuo Kakutani in 1948 with the help of Kosaku Yosida. The paper was later published in the first volume of *Proceedings of the American Mathematical Society* (No. 3, 1950). After crossing the Pacific by boat, S. Kakutani traveled by train from Seattle to Princeton with a stop in Chicago, and was carrying not only Professor Kuranishi's paper, but also a paper by Kiyoshi Oka.[16]

About the same time in Kyoto, another similar story was taking place. K. Oka wrote up the epoch-making VIIth paper on "idéal de domaines indéterminés" or the "coherence." It was difficult to read, and someone around Oka suggested handing the paper to Hideki Yukawa (physicist, Nobel Laureate, October 1949), asking him to find someone at Princeton to take care of the VIIth paper so that it would reach Henri Cartan. H. Yukawa was a graduate of Kyoto University, once in a class of K. Oka's, and was being invited to IAS Princeton 1948 (by the way, Yukawa was a professor at Columbia University, July 1949–53).

It is my guess that sometime in 1948, Kakutani and Yukawa got to know each other, and that they were both going to the IAS. Since Yukawa did not know many mathematicians, he consulted with Kakutani about Oka's paper. As a result, Kakutani took on the mission of taking care of the two papers, and carried them in his bag from Japan to the USA; this is a natural guess, because Yukawa was then professor at the University of Tokyo (1942–49). Oka VII was handed from S. Kakutani to André Weil then at Chicago, who mailed it finally to Henri Cartan (Paris); later it was published in *Bull. Soc. Math. France* **78** (1950). The scene at Chicago may be confirmed in a letter from A. Weil to H. Cartan dated 28 Sep. 1948.[17] So, the inside of Kakutani's bag in 1948 was the closest point between Professors M. Kuranishi and K. Oka. It is of note that this took place in Japan's most difficult time in the history after the surrender in 1945; there was no longer a battle, but it was still before the end of the Greater East Asian War in Japan or the Pacific War in the USA, which was settled in 1951 and effective in '52. The stories of the two papers tell us how Japanese mathematicians were making efforts to reestablish international communications after the war.

The second one is his intensive lecture course given for a week of May 2002 in Tokyo. To invite a retired professor, it was necessary by a rule of the University of Tokyo to write a special recommendation letter directly to the university president, which I wrote. The target audience was those students who were in the last year of undergraduate or graduate studies; some researchers were also in the class. The content was a discussion on Szegö kernels by means of symplectic geometry and Fourier integral operators.[18] The course was not easy for students (even researchers!) to follow, but there was quite a number of students, probably twenty or so, who remained in the last lecture on Friday afternoon. I was surprised at the end of the lecture, because the students began to clap, thanking Professor Kuranishi, then 77 years old, who looked a bit fatigued; they certainly learned something more than mathematics from his enthusiasm about mathematics. This was the only time I have ever seen such a scene.

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Takeo Ohsawa

It was in 1975 that I first encountered Masatake Kuranishi. It was when he visited Kyoto to give a series of lectures on the deformation theory of isolated singularities based on the analysis of tangential Cauchy-Riemann equations. It was shortly after I managed to pass the entrance examination for the graduate course at Kyoto University. I was attracted to mathematics by Kiyoshi Oka. He was well known in Japan, even to nonmathematicians, as a heroic figure who had solved principal questions in several complex variables. I remember that Grauert's direct image theorem, which is one of the major results after Oka's theory, stood before me like a high wall at that time, so that the idea of tangential Cauchy-Riemann equations was a kind of refreshment. Kuranishi was invited by my adviser Shigo Nakano who was a professor at RIMS (Research Institute for Mathematical Sciences). I believe that Kuranishi's visit gave me an impetus to study the $\bar{\partial}$ equations on weakly 1-complete (or pseudoconvex) manifolds.

Figure 12.

Kuranishi and Sayuri at the 1994 Osaka event honoring Masatake's 70th birthday, with T. Ohsawa, K. Takegoshi, and M. Sato in the back.

Before that period, Kuranishi was known to us as part of the drained brainpower to the US from Japan. He had impressed us with his immortal construction of versal families of deformations of compact complex manifolds. Such a family, called the Kuranishi family in Kodaira’s lecture notes “Complex manifolds” (dictated by J. Morrow), remains most important in the theory of complex manifolds. In 1974, I was surprised by Kuranishi’s unbelievable research style which was explained in an interview with Kodaira.[19]

Q. How long do you concentrate on one problem when you do not see the solution immediately?

Kodaira: I resign soon.

Q. Is it one week, two weeks, or half a year?

Kodaira: I have never continued more than half a year. It will depend on the person. You probably know Masatake Kuranishi. I once heard that he does not stop thinking until he solves the problem. He can do it for many years.

Q. Do you mean that all or nothing is his style?

Kodaira: I do not know exactly, but I was told that he proceeds as follows. Examining many possibilities at first, he continues until he is stuck at some point. Then he goes back to the beginning and repeats the same thing. He is stuck at the same place in the same way. By doing it again and again, he eventually finds something good. I cannot be so patient.

With this impression of Kuranishi in mind, I attended his lectures. Once or twice he was stuck at the blackboard. However, when he found a way out, it did not seem that he came back from the very beginning. Nevertheless, I was convinced that Kodaira was right when I saw Kuranishi dropping in Nakano's office to ask him about the proof of Dolbeault's lemma. I was there to attend a seminar and could catch what Kuranishi said. He asked Nakano, "Could you remind me how the proof of Dolbeault's lemma goes?" I was strongly impressed by this extraordinary question. After some years, it came to me that Kuranishi should have already been on the way to solving the local embedding problem of strictly pseudoconvex CR manifolds. His solution is for the manifolds of dimension ≥ 9 and appeared in magnificent papers in 1982. It was followed by a solution by Takao Akahori for the dimension ≥ 7. There exists a counterexample for the 3-dimensional case and the 5-dimensional case remains a big challenge.

Figure 13.

Kuranishi and Sayuri in the late 1990s, in front of their apartment building, and on the way to a dinner with D. H. Phong at the Japan Club.

In 1994, I saw him in Bern at a satellite conference of the ICM in Zürich. By that time he had shifted his interest to the Bergman and Szegö kernels on strongly pseudoconvex domains. He gave a comment on my talk, which I appreciated very much, and said “I want to decompose the Bergman kernel into the building blocks.” I knew that he was looking for something which lies deep in the strongly pseudoconvex case, rather than the degenerate objects without a priori symmetry, which my talk was about. In another conference, I asked him “Why are you particularly interested in the strongly pseudoconvex manifolds?” He answered “Because geometry is there.” Although Kuranishi’s words did not convert me from weakly pseudoconvex domains to strongly pseudoconvex ones, they are unforgettable and shape the outline of this outstanding mathematician.

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Duong H. Phong

It is one of the greatest privileges of my life to have been a colleague of Professor Masatake Kuranishi for almost 30 years. He was a giant of mathematics, and the testimonies in this Memorial article should give an idea of the range and depth of his legacy. But Masatake was also a model gentleman, admired and beloved by all who knew him, and who treated absolutely everyone with unfailing kindness and courtesy. In this part of the Memorial article, I would like to reminisce about some of the precious moments that he shared with me and colleagues.

I remember vividly the first time that I met Masatake. It was during the winter semester of 1975–1976, at the University of Chicago, where I was a new instructor. The winter had been dreary, and I had been really depressed. When Masatake came to give a colloquium, his talk was truly inspiring, and I eagerly joined the dinner in his honor. By pure coincidence, I happened to sit across from him, and although he did not say much, I witnessed firsthand his quiet dignity, and the inner strength that radiated from him. I could only express to myself a fervent wish to get to know him better.

This wish came true when a couple of years later, I came to Columbia University. There I got to know Masatake, and also his wonderful wife Sayuri. I sometimes marvel at the unpredictable strands of life that bring people together and exert an immense influence on their lives. I learned that in the 1960s, Masatake had offers from many prestigious mathematics departments, but he chose Columbia because Sayuri's mother, Mrs. Tsuruko Haraguchi (Arai was her maiden name), had studied psychology there, and had been the first Japanese woman to get a PhD degree from Columbia. Masatake and Sayuri welcomed visitors and young faculty to their apartment, and we experienced their warm hospitality as well as the splendid view of the Hudson River. In those days, the annual Ritt lectures at Columbia included a festive dinner where each department member would contribute his or her own dish. Masatake and Sayuri would always bring a huge and sumptuous tray of top-quality sushi, which became their trademark and was consumed by other department members in record time. Their generosity did not stop there with me. They frequently took me to the very exclusive Japan Club in New York, where Sayuri would always get me an extra take-out order of stuffed crabs, once she noticed how much I liked them. All these occasions with Masatake and Sayuri were wonderful events which remain engraved in my memory. Masatake was a true renaissance man (even his undergraduate students wrote as much in their course evaluations!), and Sayuri had very refined tastes and sometimes eccentric views that were really fun to hear. But for me, the greatest joy was just to be part of their happy life together.

These happy years passed by like a dream, marred only by a severe accident that Masatake sustained one summer in Europe. He had to undergo an operation, his return to Columbia had to be delayed, and there was concern about whether he would be able to resume teaching and research. So it was a great joy and relief for me to see Masatake back in the department within a few weeks, with barely a difference in his demeanor. He now had to walk with a cane, but he dealt with this handicap with the same equanimity and fortitude which he had shown throughout his life, for example just making sure to start a few minutes ahead of the other participants when walking to our weekly seminar dinner.

Throughout his career, Masatake attacked only deep and fundamental problems in mathematics. However, they, as well as his approaches, defy any easy classification into subfields. It appears that Masatake would identify a fundamental issue, examine it on its own merit and without any preconceptions, and then develop his own machinery to address it, undeterred by any obstacle. I recall discussing with many friends the embedding problem for CR structures that he ultimately solved. Each of us identified what we thought would be the insurmountable difficulty where we ourselves would give up. When we learned that Masatake had a solution, we thought that our diagnosis had perhaps been faulty and that these difficulties were actually not there. But it turned out that he had actually confronted them head on, and forged ahead, right through them. Thus his solution taught us more than just some new mathematical techniques or a striking new theorem. It taught us an invaluable lesson of character, perseverance, and courage. So it was always with Masatake's mathematics: deep, hard, fearless, and uncompromising in its search for the ultimate truth.

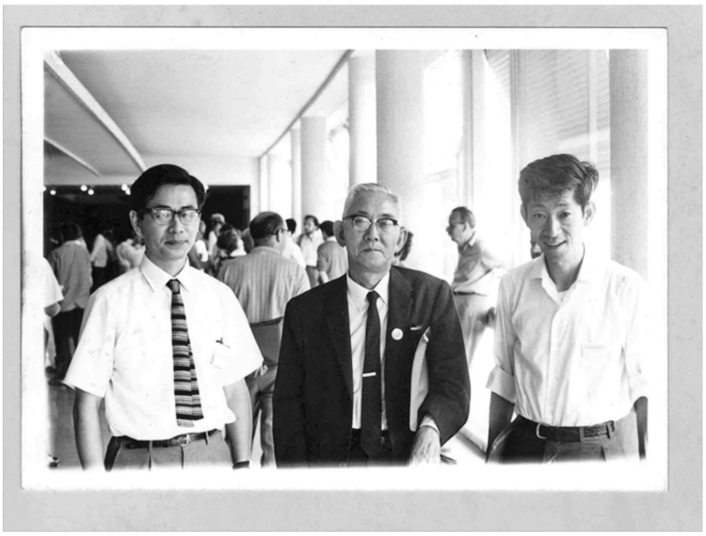
I once heard Masatake say that he drew the greatest inspiration not from mathematical conversations about specific topics, but rather from the feeling that good mathematics was being done around him. He was very tolerant and never critical of anybody, but just from his own works, one could sense that he had the most exacting standards. As a junior colleague of his, this revelation that the quality of the work around him mattered was for me a true source of motivation. Often, when I was stuck in my own research and ready to give up, I would think of Masatake and what mattered to him in order to find the courage and energy for one more effort. In this manner, in his own quiet way, he shaped the character of the Columbia Mathematics Department, and he is one of the defining figures in its history. For this, for setting an example, for his mathematical teachings, and for his personal kindness, I as well as his many former students, colleagues, and mathematical descendants are immensely grateful to him. He will forever be with us.

Mikio Sato

I am deeply saddened by the unexpected loss of Professor Masatake Kuranishi. I would like to take this opportunity to convey my sincerest condolences to his family.

Figure 14.

1970 International Congress of Mathematicians in Nice, France. From left to right: M. Sato, K. Yoshida, and M. Kuranishi.



When I first visited Columbia University, in the mid-1960s, Masatake was a professor there and he and his wife Sayuri were very kind to me. My first winter in New York, Sayuri took me to a department store to buy a thick cashmere coat. During the two years that I stayed in New York, I often talked with him about mathematics. Although I did not know enough to understand his mathematics completely, I was captivated by his deep insight and distinctive way of illustrating mathematics beautifully.

Aside from being a great mathematician, he was also a very nice and gentle person. Around the time of my retirement at RIMS in Kyoto, I was invited by Masatake to deliver a lecture course at Columbia University in the fall semester of 1992. I brought my wife and six-year-old son with me. During my visit, Masatake and Sayuri took great care of me and my family and we spent a memorable time with them. For example, they kindly arranged for my son to attend Columbia Grammar and Preparatory School. As a six-year-old, his experiences there were so special and vivid that they influenced him greatly.

Masatake was not only a dedicated mathematician who influenced many scholars, but also a very kind and humorous friend who brought joy to everyone around him. The impression he left on me, and on the mathematical community, is enormous, and he will be sorely missed.

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Yum-Tong Siu

The first time I came across the name of Masatake Kuranishi was in 1964 when I was a first-year graduate student in Minnesota, learning geometric analysis from Eugenio Calabi. At that time the theory of deformation of Kodaira-Spencer-Kuranishi was opening up a very important new direction of research. As a student I focused my study on learning the material needed to understand the theory.

Figure 15.

Y.-T. Siu with Masatake and Sayuri at the banquet of the 2005 Columbia Conference in honor of Kuranishi.

I first met Kuranishi in 1971, when I was on the faculty of Yale and was offered a professorship at Columbia. When I went to Columbia for a campus visit, Kuranishi invited me to his apartment. I discovered then that, even with his lofty intellectual stature, Kuranishi was extremely friendly and easy to talk to.

Over the years, Kuranishi and I interacted at many conferences in several complex variables. The earliest ones were the Williamstown Summer School of 1975, the Cortona Conference of 1976, and the Wisconsin Symposium of 1982. The later ones were the Oka Centennial Conference of 2001, Kuranishi's 80th Birthday Conference of 2005, and Kohn's 75th Birthday Conference of 2008. The Cortona Conference of 1976 took place in the Cortona Castle of Scuola Normale Superiore di Pisa. Both Kuranishi and I stayed in the Castle and we had a lot of time to talk in the leisurely ambience. I still vividly recall the image of Kuranishi, looking very relaxed with his corn cob pipe and at the same time very intensely immersed in his own deep mathematical thinking, when we discussed topics of common interest.

I very much admire Kuranishi's mathematics. He chose to work on important problems of great impact and introduced fundamental breakthrough techniques for their solution, persevering against great odds. His work was very thorough, always done with meticulous care. His three-part paper on the embedding of strictly pseudoconvex local CR manifolds of real dimension ≥ 9 is typical of his amazing technical prowess and the kind of attention he paid to the details in his work. All his extremely complicated computations and estimates were clearly laid out to make the line-by-line checking of his arguments possible.

For his solution of the embedding problem for a local strictly pseudoconvex CR manifold M of real dimension $2n-1 \geq 9$, the strengthened L^2 estimate of $\bar{\partial}_b$ on a strictly pseudoconvex local hypersurface X in $\mathbb{C}^n$ is first obtained by marking out an open neighborhood W of the reference point in X with the use of a distance function t which is the real part of a local holomorphic function. The real part of a local holomorphic function is used to avoid any additional pseudoconvexity property of the boundary of W when the estimate is to be established on W. For a local basis $Y_1, \cdots, Y_{n-1}$ of $(1,0)$ vectors, the multiplier

$$\frac{1}{\sqrt{\sum_{j=1}^{n-1} |Y_j t|^2}}$$

is introduced to strengthen the L^2 estimate. One then starts out with a good local smooth non-$\bar{\partial}_b$-closed diffeomorphism f of M onto X. Through f, the solution of $\bar{\partial}_b$ on X with the strengthened estimate provides a way of approximately solving the $\bar{\partial}_b$ equation on M which is then used to modify f to make it closer to being $\bar{\partial}_b$-closed. The process is iterated to yield a $\bar{\partial}_b$-closed embedding of M into $\mathbb{C}^n$. His ideas for the solution are ingenious and completely unexpected. The task of actually working out the details is formidably demanding.

Kuranishi was a perfect gentleman-scholar in the Asian tradition. With his calm and unassuming demeanor, he put people at ease, was always encouraging and inspiring, and stood ready to help. He will be sorely missed. His life and work will always stay fondly in our memory.

Shing-Tung Yau

I knew Masatake Kuranishi for more than forty-five years. I believe the first time I met him was in 1975 at the conference on several complex variables in Williamstown, Massachusetts. He was already a well-established mathematician, while I had just graduated not long ago. But of course, I learned his fundamental contribution on the moduli space on complex structures and Cartan's theory of prolongation on exterior differential systems when I was in graduate school. I was somewhat surprised to find out that he was a very humble gentleman. I spent some time at the Courant Institute at NYU right after that conference. And occasionally I went to Columbia and met him there.

Figure 16.

Kuranishi and S.-T. Yau.

When I was a faculty member at the IAS in the 1980s, we ran a special year in several complex variables. We were excited to study his spectacular achievements on CR embedding, a truly deep work in analysis. But I got to know Professor Kuranishi better when I visited Columbia in 1999 as the Eilenberg Professor. I met him much more frequently. He was always rather quiet while smiling frequently. Columbia offered me a job and Professor Kuranishi entertained me with very nice Japanese food in a great Japanese restaurant owned by his good friend. He also invited my wife and me to his beautiful home. I was very touched by his friendship.

He submitted a paper to the *Journal of Differential Geometry* where he presented a proof of the Hopf conjecture in which the product of two spheres admits a metric with positive sectional curvature. This took me by surprise. The paper was over 150 pages. Despite it being full of original ideas, it fell through eventually. But he treated the whole process in the most graceful way, and I am grateful for it.

I met him several times since he retired to Japan about ten years ago. The last time was when I gave the Takagi Lecture at the University of Tokyo. He came to my lecture even though he was about 86, and we had nice conversations. When I learned from Phong that he passed away, I was taken by surprise, as I always had a good picture of him in a very healthy manner, although he walked slowly. Mathematicians will always remember his great contribution and I will miss this excellent role model.

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