$$
\begin{aligned}
\mathcal{P}\{a\} &= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ (\mathcal{P}\{a\}) \setminus \{\emptyset\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} x \notin \{\emptyset\} \\ x \in \mathcal{P}\{a\} \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} \neg x \in \{\emptyset\} \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} \neg x = \emptyset \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} \exists b b \in x \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \exists b \left(\bigwedge\left\{ \begin{array}{c} b \in x \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \exists b \left(\bigwedge\left\{ \begin{array}{c} b \in x \\ b \in x \implies b \in \{a\} \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \exists b \left(\bigwedge\left\{ \begin{array}{c} b \in x \\ (\neg \bot) \implies b \in \{a\} \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \exists b \left(\bigwedge\left\{ \begin{array}{c} b \in x \\ b \in \{a\} \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \exists b \left(\bigwedge\left\{ \begin{array}{c} b \in x \\ b = a \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \exists b \left(\bigwedge\left\{ \begin{array}{c} a \in x \\ b = a \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} a \in x \\ \exists b b = a \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} a \in x \\ \neg \bot \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} a \in x \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} \forall z \neg \left(\bigwedge\left\{ \begin{array}{c} z \in \{a\} \\ \neg z \in x \end{array} \right\}\right) \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \left(\bigwedge\left\{ \begin{array}{c} \forall z\,(z \in \{a\} \implies z \in x) \\ \forall z\,(z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \left\{x \middle| \forall z \left(\bigwedge\left\{ \begin{array}{c} (z \in \{a\} \implies z \in x) \\ (z \in x \implies z \in \{a\}) \end{array} \right\}\right)\right\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \{x \mid \forall z\,(z \in \{a\} \iff z \in x)\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \{x \mid \{a\} = x\} \end{array} \right\} \\
&= \bigcup\left\{ \begin{array}{c} \{\emptyset\} \\ \{\{a\}\} \end{array} \right\} \\
&= \{\emptyset; \{a\}\}
\end{aligned}
$$