

Pas de titre

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Exercice 0.1 ★ **Pas de titre**
Déterminer, en utilisant des développements limités, les limites suivantes :

$$1. \lim_{x \rightarrow +\infty} \sqrt{x^2 + 3x + 2} - x$$

$$4. \lim_{x \rightarrow 0} \frac{x(1 + \cos x) - 2 \tan x}{2x - \sin x - \tan x}$$

$$2. \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$$

$$5. \lim_{x \rightarrow 0} \frac{e^x - x - \cos x}{x^2}$$

$$3. \lim_{x \rightarrow 0} \frac{\sin(x - \sin x)}{\sqrt{1 + x^3} - 1}$$

$$6. \lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{\ln(1 + x)}$$

Solution :

1.

$$\begin{aligned} \sqrt{x^2 + 3x + 2} - x &= x \left(\sqrt{1 + \frac{3}{x} + \frac{2}{x^2}} - 1 \right) \\ &\stackrel{X=\frac{1}{x}}{=} \frac{\sqrt{1 + 3X + 2X^2} - 1}{X} \\ &= \frac{\frac{3}{2}X + \underset{X \rightarrow 0}{o}(X)}{X} \\ &= \frac{3}{2} + \underset{X \rightarrow 0}{o}(1) \xrightarrow[X \rightarrow 0]{} \boxed{\frac{3}{2}} \end{aligned}$$

2.

$$\begin{aligned}
\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} &= e^{\frac{\ln\left(\frac{\sin x}{x}\right)}{x^2}} \\
&= e^{\frac{\ln\left(1 - \frac{1}{6}x^2 + o_{x \rightarrow 0}(x^2)\right)}{x^2}} \\
&= e^{\frac{-\frac{1}{6}x^2 + o_{x \rightarrow 0}(x^2)}{x^2}} \\
&= e^{-\frac{1}{6} + \underset{x \rightarrow 0}{o}(1)} \xrightarrow{x \rightarrow 0} \boxed{e^{-\frac{1}{6}}}
\end{aligned}$$

3.

$$\begin{aligned}
\frac{\sin(x - \sin x)}{\sqrt{1 + x^3} - 1} &= \frac{\sin\left(\frac{1}{6}x^3 + o_{x \rightarrow 0}(x^3)\right)}{\frac{1}{2}x^3 + o_{x \rightarrow 0}(x^3)} \\
&= \frac{\frac{1}{6}x^3 + o_{x \rightarrow 0}(x^3)}{\frac{1}{2}x^3 + o_{x \rightarrow 0}(x^3)} \\
&= \frac{\frac{1}{6} + \underset{x \rightarrow 0}{o}(1)}{\frac{1}{2} + \underset{x \rightarrow 0}{o}(1)} \xrightarrow{x \rightarrow 0} \boxed{\frac{1}{3}}
\end{aligned}$$

4.

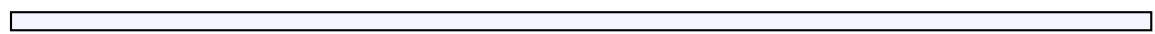
$$\begin{aligned}
\frac{x(1 + \cos x) - 2\tan x}{2x - \sin x - \tan x} &= \frac{-\frac{7}{6}x^3 + o_{x \rightarrow 0}(x^3)}{-\frac{1}{6}x^3 + o_{x \rightarrow 0}(x^3)} \\
&= \frac{-\frac{7}{6} + \underset{x \rightarrow 0}{o}(1)}{-\frac{1}{6} + \underset{x \rightarrow 0}{o}(1)} \xrightarrow{x \rightarrow 0} \boxed{7}
\end{aligned}$$

5.

$$\begin{aligned}
\frac{e^x - x - \cos x}{x^2} &= \frac{x^2 + o_{x \rightarrow 0}(x^2)}{x^2} \\
&= 1 + \underset{x \rightarrow 0}{o}(1) \xrightarrow{x \rightarrow 0} \boxed{1}
\end{aligned}$$

6.

$$\begin{aligned}
\frac{1}{x} - \frac{1}{\ln(1 + x)} &= \frac{\ln(1 + x) - x}{x \ln(1 + x)} \\
&\underset{x \rightarrow 0}{\sim} \frac{\ln(1 + x) - x}{x^2} \\
&= \frac{-\frac{1}{2}x^2 + o_{x \rightarrow 0}(x^2)}{x^2} \\
&= -\frac{1}{2} + \underset{x \rightarrow 0}{o}(1) \xrightarrow{x \rightarrow 0} \boxed{-\frac{1}{2}}
\end{aligned}$$



Références