Calcul, divers

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Exercice 0.1 ★★ ***Calcul, divers***

Prouver la convergence des intégrales suivantes puis les calculer :

1. $\int_{t=2}^{+\infty} \dfrac{e^t\,dt}{(e^{2t}-5e^t+6)(e^t-1)}$
2. $\int_{t=0}^{+\infty} \dfrac{dt}{\operatorname{ch}^4 t+\operatorname{sh}^4 t}$
3. $\int_{t=0}^{+\infty} te^{-\sqrt{t}}\,dt$
4. $\int_{t=0}^{1} \arcsin t\,dt$
5. $\int_{t=0}^{1} \dfrac{\ln(1-t^2)}{t^2}\,dt$
6. $\int_{t=0}^{+\infty} \dfrac{t^3\ln t}{(1+t^4)^3}\,dt$
7. $\int_{t=0}^{\pi/2} \ln\sin t\,dt$
8. $\int_{t=0}^{1} \dfrac{\ln t}{\sqrt{1-t}}\,dt$
9. $\int_{t=0}^{+\infty} \dfrac{\ln t}{1+t^2}\,dt$
10. $\int_{t=0}^{1} \dfrac{\ln t}{(1+t)\sqrt{1-t^2}}\,dt$
11. $\int_{t=0}^{1} \dfrac{dt}{\sqrt{1+t}+\sqrt{1-t}}$
12. $\int_{t=0}^{+\infty} \ln\Big(1+\dfrac{a^2}{t^2}\Big)\,dt$
13. $\int_{t=0}^{+\infty} \ln\left|\dfrac{1+t}{1-t}\right| \dfrac{t\,dt}{(a^2+t^2)^2}$

Solution :

1. $\int_{t=2}^{+\infty} \dfrac{e^t\,dt}{(e^{2t}-5e^t+6)(e^t-1)}$ $\ln(\frac{e^2-2}{\sqrt{e^4-4e^2+3}})$

2. $\int_{t=0}^{+\infty} \dfrac{dt}{\operatorname{ch}^4 t + \operatorname{sh}^4 t}$ $\frac{1}{\sqrt{2}} \ln(\sqrt{2}+1)$

3. $\int_{t=0}^{+\infty} te^{-\sqrt{t}}\,dt$ 12

4. $\int_{t=0}^{1} \arcsin t\,dt$ $\frac{\pi}{2} - 1$

5. $\int_{t=0}^{1} \dfrac{\ln(1-t^2)}{t^2}\,dt$ $-2\ln 2$

6. $\int_{t=0}^{+\infty} \dfrac{t^3 \ln t}{(1+t^4)^3}\,dt$ $-\frac{1}{32}$

7. $\int_{t=0}^{\pi/2} \ln \sin t\,dt$ $-\frac{\pi \ln 2}{2}$

8. $\int_{t=0}^{1} \dfrac{\ln t}{\sqrt{1-t}}\,dt$ $4\ln 2 - 4$ $(u = \sqrt{1-t})$

9. $\int_{t=0}^{+\infty} \dfrac{\ln t}{1+t^2}\,dt$ 0 $(u = 1/t)$

10. $\int_{t=0}^{1} \dfrac{\ln t}{(1+t)\sqrt{1-t^2}}\,dt$ $\ln 2 - \frac{\pi}{2}$ $(u = \sqrt{(1-t)/(1+t)})$

11. $\int_{t=0}^{1} \dfrac{dt}{\sqrt{1+t}+\sqrt{1-t}}$ $\sqrt{2} + \ln(\sqrt{2}-1)$

12. $\int_{t=0}^{+\infty} \ln\Big(1 + \dfrac{a^2}{t^2}\Big)\,dt$ $\pi|a|$

13. $\int_{t=0}^{+\infty} \ln\left|\dfrac{1+t}{1-t}\right| \dfrac{t\,dt}{(a^2+t^2)^2}$ $\dfrac{\pi}{2|a|(a^2+1)}$

Références