

Calcul, radicaux

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Exercice 0.1 ★★ Calcul, radicaux

Prouver la convergence des intégrales suivantes puis les calculer :

1. $\int_{t=0}^1 \sqrt{\frac{t}{1-t}} dt$
2. $\int_{t=1}^{10} \frac{dt}{\sqrt[3]{t-2}}$
3. $\int_{t=a}^b \frac{dt}{\sqrt{(t-a)(b-t)}}$
4. $\int_{t=0}^1 \frac{t^5 dt}{\sqrt{1-t^2}}$
5. $\int_{t=-1}^1 \frac{dt}{(1+t^2)\sqrt{1-t^2}}$
6. $\int_{t=0}^1 \frac{dt}{(4-t^2)\sqrt{1-t^2}}$
7. $\int_{t=0}^1 \frac{t dt}{\sqrt{(1-t)(1+3t)}}$
8. $\int_{t=0}^1 \frac{dt}{(1+t)\sqrt{t^2-t^3}}$
9. $\int_{t=0}^1 \arctan \sqrt{1-t^2} dt$
10. $\int_{t=1}^{+\infty} \frac{dt}{t\sqrt{t^{10}+t^5+1}}$
11. $\int_{t=0}^{+\infty} \frac{dt}{(1+t^2)\sqrt{t}}$

Solution :

1. $\int_{t=0}^1 \sqrt{\frac{t}{1-t}} dt \frac{\pi}{2}$
2. $\int_{t=1}^{10} \frac{dt}{\sqrt[3]{t-2}} \frac{9}{2}$

3. $\int_{t=a}^b \frac{dt}{\sqrt{(t-a)(b-t)}} \pi$
4. $\int_{t=0}^1 \frac{t^5 dt}{\sqrt{1-t^2}} \frac{8}{15}$
5. $\int_{t=-1}^1 \frac{dt}{(1+t^2)\sqrt{1-t^2}} \frac{\pi}{\sqrt{2}}$
6. $\int_{t=0}^1 \frac{dt}{(4-t^2)\sqrt{1-t^2}} \frac{\pi}{4\sqrt{3}}$
7. $\int_{t=0}^1 \frac{t dt}{\sqrt{(1-t)(1+3t)}} \frac{2\pi}{9\sqrt{3}} + \frac{1}{3}$
8. $\int_{t=0}^1 \frac{dt}{(1+t)\sqrt[3]{t^2-t^3}} \frac{\pi\sqrt[3]{4}}{\sqrt{3}}$
9. $\int_{t=0}^1 \arctan \sqrt{1-t^2} dt \frac{\pi(\sqrt{2}-1)}{2}$
10. $\int_{t=1}^{+\infty} \frac{dt}{t\sqrt{t^{10}+t^5+1}} \frac{1}{5} \ln\left(1 + \frac{2}{\sqrt{3}}\right)$
11. $\int_{t=0}^{+\infty} \frac{dt}{(1+t^2)\sqrt{t}} \frac{\pi}{\sqrt{2}}$

Références