

Calcul, fractions rationnelles

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Exercice 0.1 ★★ Calcul, fractions rationnelles

Prouver la convergence des intégrales suivantes puis les calculer :

1. $\int_{t=0}^{+\infty} \frac{dt}{(1+t^2)^2}$
2. $\int_{t=-\infty}^{+\infty} \frac{dt}{t^2+2t+2}$
3. $\int_{t=0}^{+\infty} \frac{dt}{(1+t^2)^4}$
4. $\int_{t=-\infty}^{+\infty} \frac{dt}{(t^2+1)(t^2-2t\cos\alpha+1)}$
5. $\int_{t=0}^{+\infty} \frac{2t^2+1}{(t^2+1)^2} dt$
6. $\int_{t=-\infty}^{+\infty} \frac{t^2 dt}{(t^2+1)(t^2+a^2)}$
7. $\int_{t=0}^{+\infty} \frac{dt}{1+t^4}$
8. $\int_{t=0}^{+\infty} \frac{t^2 dt}{1+t^4}$
9. $\int_{t=1}^{+\infty} \frac{dt}{t^6(1+t^{10})}$

Solution :

1. $\int_{t=0}^{+\infty} \frac{dt}{(1+t^2)^2} = \frac{\pi}{4}$
2. $\int_{t=-\infty}^{+\infty} \frac{dt}{t^2+2t+2} = \pi$
3. $\int_{t=0}^{+\infty} \frac{dt}{(1+t^2)^4} = \frac{5\pi}{32}$
4. $\int_{t=-\infty}^{+\infty} \frac{dt}{(t^2+1)(t^2-2t\cos\alpha+1)} = \frac{\pi}{2|\sin\alpha|}$

$$5. \int_{t=0}^{+\infty} \frac{2t^2 + 1}{(t^2 + 1)^2} dt \quad \frac{3\pi}{4}$$

$$6. \int_{t=-\infty}^{+\infty} \frac{t^2 dt}{(t^2 + 1)(t^2 + a^2)} \quad \frac{\pi}{1+|a|}$$

$$7. \int_{t=0}^{+\infty} \frac{dt}{1 + t^4} \quad \frac{\pi}{2\sqrt{2}}$$

$$8. \int_{t=0}^{+\infty} \frac{t^2 dt}{1 + t^4} \quad \frac{\pi}{2\sqrt{2}}$$

$$9. \int_{t=1}^{+\infty} \frac{dt}{t^6(1 + t^{10})} \quad \frac{4-\pi}{20}$$

Références