

$$\int_0^1 \left( \int_0^1 f(x, y) dx \right)^2 dy + \int_0^1 \left( \int_0^1 f(x, y) dy \right)^2 dx \leq \left( \int_0^1 \int_0^1 f(x, y) dx dy \right)^2 + \int_0^1 \int_0^1 f(x, y)^2 dx dy$$

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**Exercice 0.1** ★  $\int_0^1 \left( \int_0^1 f(x, y) dx \right)^2 dy + \int_0^1 \left( \int_0^1 f(x, y) dy \right)^2 dx \leq \left( \int_0^1 \int_0^1 f(x, y) dx dy \right)^2 + \int_0^1 \int_0^1 f(x, y)^2 dx dy$

Putnam (2005), [1] 2005/8.

Soit  $f \in \mathcal{C}^0([0, 1] \times [0, 1], \mathbb{R})$ , montrer que

$$\begin{aligned} \int_0^1 \left( \int_0^1 f(x, y) dx \right)^2 dy + \int_0^1 \left( \int_0^1 f(x, y) dy \right)^2 dx \\ \leq \left( \int_0^1 \int_0^1 f(x, y) dx dy \right)^2 + \int_0^1 \int_0^1 f(x, y)^2 dx dy. \end{aligned}$$

## Références

[1] American Mathematical Monthly. M.A.A., maa@?????.fr.